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May 15, 2008

Journal of Fusion Energy

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Abstract

It is shown that spheromak equilibria, stable at zero-beta but departing from the Taylor state, could be sustained by non-inductive current drive at acceptable power levels. Stability to both ideal MHD and tearing modes is verified using the NIMROD code for linear stability analysis. Non-linear NIMROD calculations with non-inductive current drive and pressure effects could point the way to improved fusion reactors.

1. Introduction

Building on earlier results in CTX [1], remarkably stable spheromak equilibria confining plasmas at electron temperatures up to 500 eV have now been achieved in the SSPX experiment using helicity injection by an electrostatic gun [2], but only transiently, after the gun current is reduced below the level required to drive helicity injection by magnetic relaxation as discussed by Taylor [3]. Sustaining high temperatures in steady state by helicity injection, the basis for earlier spheromak fusion reactor studies [4] has proved elusive due to continuing magnetic turbulence that cools the plasma during helicity injection [5], perhaps for fundamental reasons [6].

Here we discuss an alternative approach in which non-inductive current drive maintains a stable state, feasible only for equilibria that depart significantly from the Taylor state that would have large ohmic losses on open field lines and near the edge inside the separatrix, as in the stable state of SSPX. Stability is characterized by $\lambda = \mu_o(\mathbf{j}\cdot\mathbf{B}/B^2)$ for magnetic field **B**. As with the Taylor state with constant λ, stability depends on flattening the λ profile, but only in the interior, allowing both λ and j to fall to zero at the edge. The existence of such states in spheromaks, if not well known is also not unexpected based on results in the literature. Specifically, our work was suggested by similar work of Robinson for tearing modes in reversed field pinches [7]. It is also

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known that pressure may drive instability in a spheromak stable to tearing, a point we will return to below.

The reader may ask, if theory predicts tearing-stable profiles, why both RFP's and spheromak experiments have exhibited tearing of flux surfaces detrimental to good confinement. The reason is that the flattened λ profile required for stability is not a natural state, since in the absence of instability non-uniform resistivity tends to create gradients in λ. Hence stability requires control of the λ profile, already accomplished transiently in RFP experiments with significant improvements in energy confinement and temperature [8]. Non-inductive current drive could do this in steady state.

The main purpose of this paper is to present preliminary stability calculations using the NIMROD code [9] to analyze linear stability, as motivation for future work on NIMROD to extend our work to the non-linear regime, including pressure driven modes.

2. Stable Equilibria

To seek stable equilibria, we take as our model:

$$\lambda(\psi) = \lambda_o[1 - (\psi/\psi_E)^N] \quad , \qquad \lambda = 0 \text{ for } 0 < \psi < \psi_E \qquad (1)$$

where ψ is the poloidal magnetic flux function with $\psi = \psi_E$ at the plasma edge. The Grad-Shafranov equilibrium equation is solved using the Corsica code with SSPX flux conserver geometry including an electrostatic gun with poloidal field coils producing a bias flux ψ_E on open field lines [10, 11]. Here the gun voltage generating current in SSPX is set equal to zero, represented in Eq. (1) by $\lambda = 0$ for $0 < \psi < \psi_E$ (taken positive). While we wish to specify $\lambda(\psi)$, the actual inputs to Corsica are the plasma pressure $p(\psi)$ and $F(\psi) = RB_\phi$, or equivalently $dF/d\psi$ which is exactly $\lambda(\psi)$ at zero pressure. Following Ref. [10], we in fact define $dF/d\psi = \lambda(\psi)$ in Eq. (1) with little error for the small but non-zero pressure included in our calculations.

The shape of the λ profile is controlled by N in Eq. (1). An example stable equilibrium with N = 6 is shown in Fig. 1 giving profiles for λ and j, and also the safety factor $q(\psi)$ prominent in tokamak theory. Note that j is zero at the plasma edge; note also the flattened $\langle\lambda\rangle$ profile. Fig. 2 displays the closed flux surfaces, λ being zero outside the last closed surface. The λ and j plotted here are flux surface averages $<\lambda> = \mu_o(<\mathbf{j}\cdot\mathbf{B}>/<B^2>)$ and $<j> = (<\mathbf{j}\cdot\mathbf{B}>/<B^2>^{1/2})$.

3. Stability Calculations

Turning to stability, we note first that kink-like instabilities driven by gun current in SSPX – the dominant process in helicity injection – should not occur for our equilibria with zero gun current. Then the only likely zero-beta modes are internal modes as if the plasma boundary were rigid, together with tilt and shift modes corresponding to rigid rotations or translations. Stability of the equilibrium in Fig. 1 to tilt and shift modes has been verified using the DCON code [12].

Before discussing NIMROD results for internal modes, we note that analytical insight as to why merely flattening λ in the interior might be sufficient to stabilize tearing is given by an approximate formula for the free energy parameter Δ' (the quantity calculated by Robinson [7]), yielding a well-known stability criterion for tokamaks [13]. Adapted to spheromaks, this gives stability if:

$$\left|aq^2(\lambda'/q')\right| \qquad < \qquad m \tag{2}$$

where m is the poloidal mode number and (λ'/q') with $' \equiv d/dr$ is the main factor in the destabilizing term of the free energy $\delta W \propto -r\Delta'$. Stability of λ profiles like that in Fig. 1 would seem to follow from flat λ in the interior and the small value of q at the edge. To evaluate the criterion, we have derived q from λ in the usual way, giving with reasonable analytical approximations instability for resonances at $r = 0$ if $N = 1$ and stability if $N \geq 2$, and with a little more work the same result at all r. Pearlstein has calculated Δ' exactly for the cylinder model of a spheromak and finds stability if $N > 5$ [14]. But toroidal effects missing in cylinder models are important for spheromaks. Quantitative guidance requires further numerical computation, to which we now turn, using NIMROD that also takes account of toroidal effects.

NIMROD is a non-linear resistive MHD code evolving initial states in time in 3D [9]. Here we use this code only to determine linear stability. To do so, the 2D equilibria calculated using the Corsica code, as discussed above, are accurately introduced into the

NIMROD code. Then linear stability (of both ideal MHD and tearing modes) is tested by time-dependent calculations with non-linear terms disabled, to determine if initial perturbations grow in time. The standard NIMROD numerical techniques yielding reasonable agreement with modes observed in SSPX are employed. We select out internal modes by imposing boundary conditions representing a conducting wall at the plasma boundary coinciding with the last closed flux surface (see Fig. 2). NIMROD tests both ideal MHD and resistive tearing modes. Stability is investigated for values of N = 2 to 6 in Eq. (1) in order to detect numerically the threshold value of N above which stability is obtained. Modes with toroidal mode numbers n up to 10 were investigated.

Fig. 3 shows the λ and q profiles for N = 2, 3 and 4 (see Fig. 1 for N = 6 profiles). In NIMROD calculations, the modes with toroidal mode numbers n = 4 and n = 5 had positive growth rate for N = 2. The growth rate of the perturbation energy is shown in Fig. 4. (Note that the N = 2 case does not have a q = 1/3 surface to support a n = 3 mode.) For N = 3, the n = 3 mode is unstable and the linear growth rate is also shown in Figure 4. In contrast to these cases, for N = 4 all modes are observed to be stable. This is illustrated in Fig. 5 by the negative growth rate for n = 3, 4 and 5. Other mode numbers also give negative growth rate for N = 4. These calculations were repeated for N = 5 and N = 6 and these also show stability to all modes up to n = 10.

Thus NIMROD confirms our expectation that sufficiently flat λ profiles are stable to current-driven ideal and resistive internal modes. For the equilibrium model of Eq. (1), the threshold value for stability as indicated by NIMROD is $3 \leq N \leq 4$, indicating less flattening to achieve stability than did Pearlstein's cylinder model.

4. Non-Inductive Current Drive

To assess the relevance of our work to fusion energy research, we consider specifically neutral beam current drive and calculate the beam power P_{CD} required to sustain the current, using the model in Ref. [15] to obtain finally:

$$P_{CD} = \int dV\, SE = 20\,(InR/T)C \qquad (3)$$

where $S \propto I$ is the beam deposition rate, power is in MW for toroidal current I in MA with density n in units 10^{20} m^{-3} and electron temperature T in KeV, and C is a weighting factor including effects of beam orientation relative to field lines. Details are given in Ref. [16]. Eq. (3) exhibits the usual scalings for non-inductive current drive power and it fits calculations for spheromaks in Ref. [17], and also results from the DIIID tokamak [18] when corrected to take into account non-optimum beam energy per the model of Ref. [15]; limited access for beam injection parallel to field lines in the DIIID tokamak; and a factor 2 degradation due to instability driven by super-Alfvenic beam ions in these experiments, giving altogether $C \approx 10$ [19].

As a figure of merit, we use Eq. (3) to calculate the fusion power gain $Q \propto (4\pi R^2 P_W/P_{CD})$ with wall load P_W due to fusion neutrons bombarding a spherical vacuum vessel of radius R that also serves as the flux conserver. Using Eq. (3) and calculating P_W using known nuclear cross sections yields, after a little algebra, the value of R required to achieve a given Q at a wall load P_W is [16]:

$$R = 0.09\, P_W^{-1/3} (IQ)^{2/3} \tag{4}$$

where we have shown that for optimized injection the weighting factor C and one appearing in the integral to obtain P_W roughly cancel for the profiles of Fig. 1. Guessing I = 50 MA for ignition (the value in Ref. [4]), Q = 20 gives R = 3 m for $P_W = 20$ MW/m^2 (the parameters of Ref. [4]) and R = 5 m at $P_W = 5$ MW/m^2 as in most reactor studies, where, as in Eq. (3), R is the size of the vessel (not the plasma major radius $\approx 1/2$ R). This is to be compared with an equivalent R = 20 m for ITER and R = 10 m for the ARIES-AT advanced tokamak reactor [20]. The smaller reactor has reduced power and lower β.

The actual current required for ignition is not known for spheromaks. Assuming ideal and tearing MHD stability, we have attempted to extrapolate tokamak scalings to spheromaks, to represent non-MHD processes. ITER-98(y,2) scaling has been examined in Ref. [19], with the often-made assumption that the power P in this formula should be interpreted as $P = \int 3nT/\tau_E$, yielding:

$$n\tau_E = 3.8 \times 10^{-3}\, (I^{3.0} A^{2.25}/T^{2.2})(n^{0.1}/a^{0.3}) \tag{5}$$

with aspect ratio A. This scaling together with heat balance gives the temperature to be used in Eq. (3) [19], as discussed in Section 5. Eq. (5) gives ignition for the reactor cases cited above for small A = 1.5, and for A = 1 at somewhat higher current. Like tokamaks, stabilized RFP's with much higher A may require less current but overall larger dimensions and larger fusion power for a given wall load. It can be shown that a similar extrapolation of L-mode scaling gives results consistent with SSPX [21].

5. Discussion

The possibility of spheromak reactors smaller than tokamaks, with no toroidal coils, continues to offer an attractive alternative route to fusion power, if the good plasma confinement exhibited in SSPX extrapolates to larger systems. The confidence to pursue a spheromak research program with current drive could be greatly strengthened by more computer simulations, using the NIMROD code already validated extensively to understand magnetic turbulence in SSPX [5].

As mentioned in the Introduction, an unresolved physics issue concerns effects at finite β. Experiments in SSPX have already achieved a peak electron $\beta \approx 10\%$, comparable to that in the reactor design of Ref. [4]. However, these results were achieved transiently, during resistive decay of the field after helicity injection ceases. For steady state, theory has long predicted the slow growth of magnetic islands on this timescale in spheromaks stable to tearing modes, possibly giving large saturated island widths of order $w/a \propto (\beta/|\Delta' a|)$ [22]. Since they are always linearly unstable, an evaluation of these pressure-driven modes requires non-linear dynamics. The non-linear NIMROD simulations advocated here could explore pressure-driven resistive instabilities as well as tearing, in the actual low-aspect-ratio spheromak geometry.

Calculations for neutral beam injection experiments that could study beta effects in SSPX are discussed in Ref. [23]. In these calculations the NFREYA beam deposition package is applied to Corsica-generated target plasmas with electron temperatures up to 360 eV. At an injection power of 1.5 MW for 2 ms, beam injection dominates over ohmic heating if beams are directed onto the core region near the magnetic axis, and q profiles are altered by the beams. We have applied this code at higher beam power to show that spreading beam injection angles should be able to control the q and λ profiles in plasmas several ion orbits across, as proposed in this paper.

Preliminary ideas are discussed in Ref. [19] for a sequence of new spheromak experiments using neutral beams both to build up the current and to maintain profile control during the buildup. Cases are given for a “proof of principle” experiment with 40 KeV beams and a plasma minor radius a = 0.25m, and a device achieving ignition with 80 KeV beams and a = 0.75m, all at $\beta < 1\%$. While much more work is needed to produce believable numbers, these results suggest that detailed work might produce interesting results. For the cases above, L-mode scaling was assumed during tearing-stable buildup, and Eq. (5) in steady state [19], as discussed in Section 4. Buildup could be initiated on a gun-created target plasma, as in SSPX. The main cost for an experimental program to pursue these ideas could be avoided by sharing existing neutral beam systems employed in tokamak research.

Historically, it was the ideal and resistive MHD physics featured in NIMROD that drove the worldwide fusion program toward tokamaks. NIMROD simulations extending our work could help decide whether the toroidal field coils of tokamaks are really necessary for stability, perhaps paving the way to better fusion reactors in the future, in parallel with ITER.

Acknowledgements

The authors wish to thank D. Brennan, B. I. Cohen, E. B. Hooper, L. L. Lodestro and C. R. Sovinec for many helpful discussions. We especially thank L. D. Pearlstein for his unpublished calculations of Δ' for the cylinder approximation of a spheromak that helped motivate this work. This work was supported in part by the U. S. Department of Energy under contracts W-7405-ENG-48 and DE-AC52-07NA27344 at the Lawrence Livermore National Laboratory.

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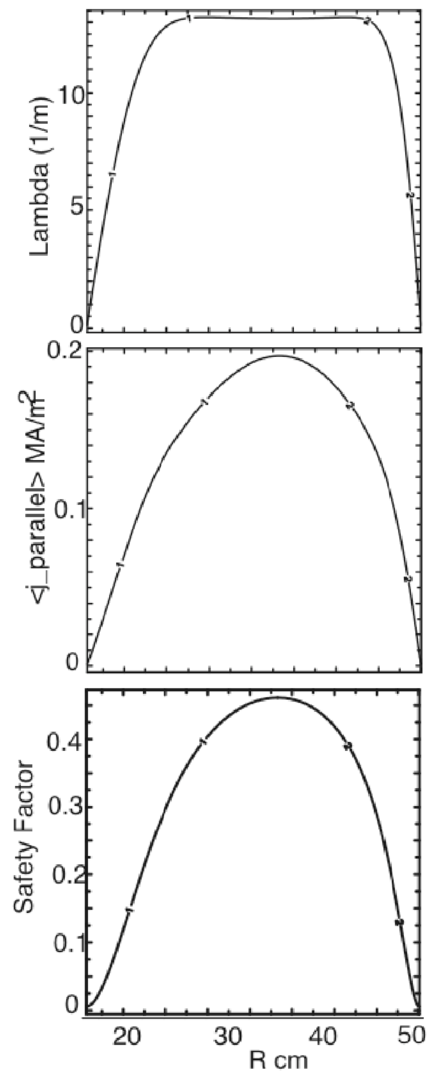

Fig. 1 Profiles for exponent N=6 (Eq. (1))

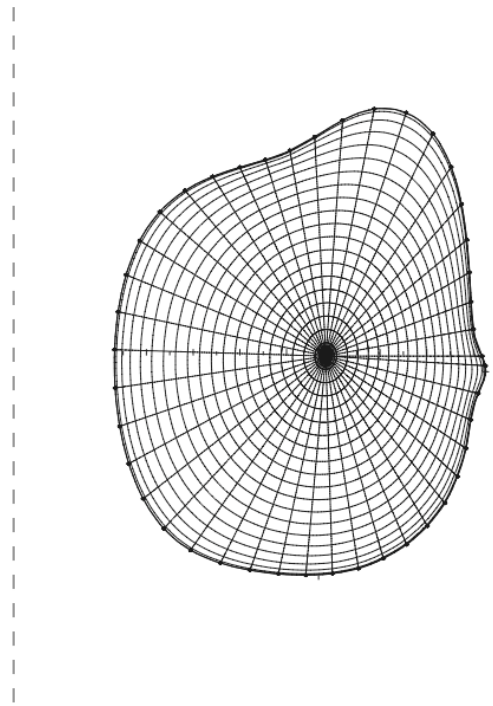

Fig. 2 Poloidal flux contours for profile shown in Fig. 2 (N=6)

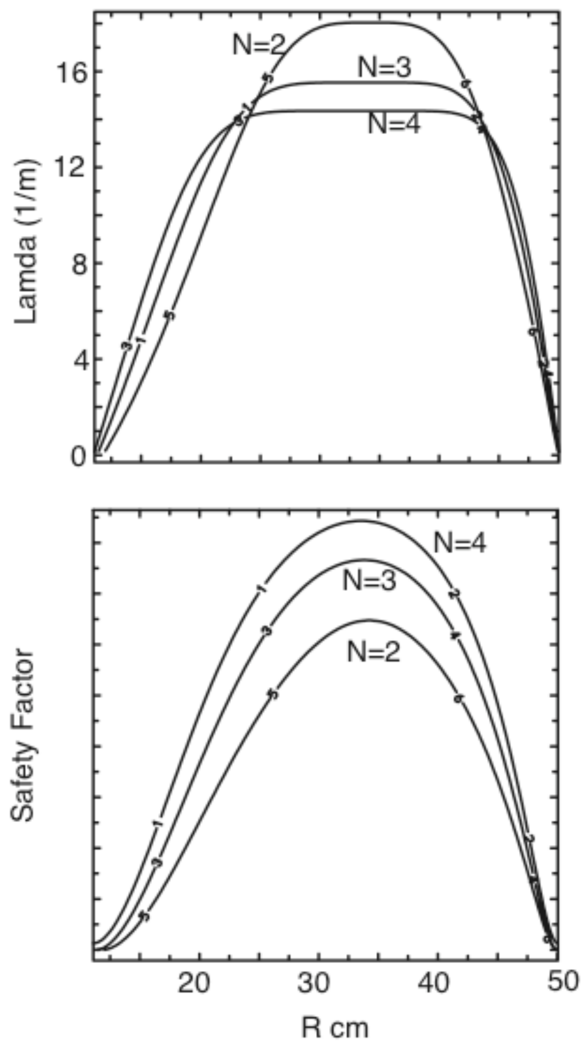

Fig.3 λ and q profiles for N=2,3 and 4; for N=2 the maximum value of q is 0.321.

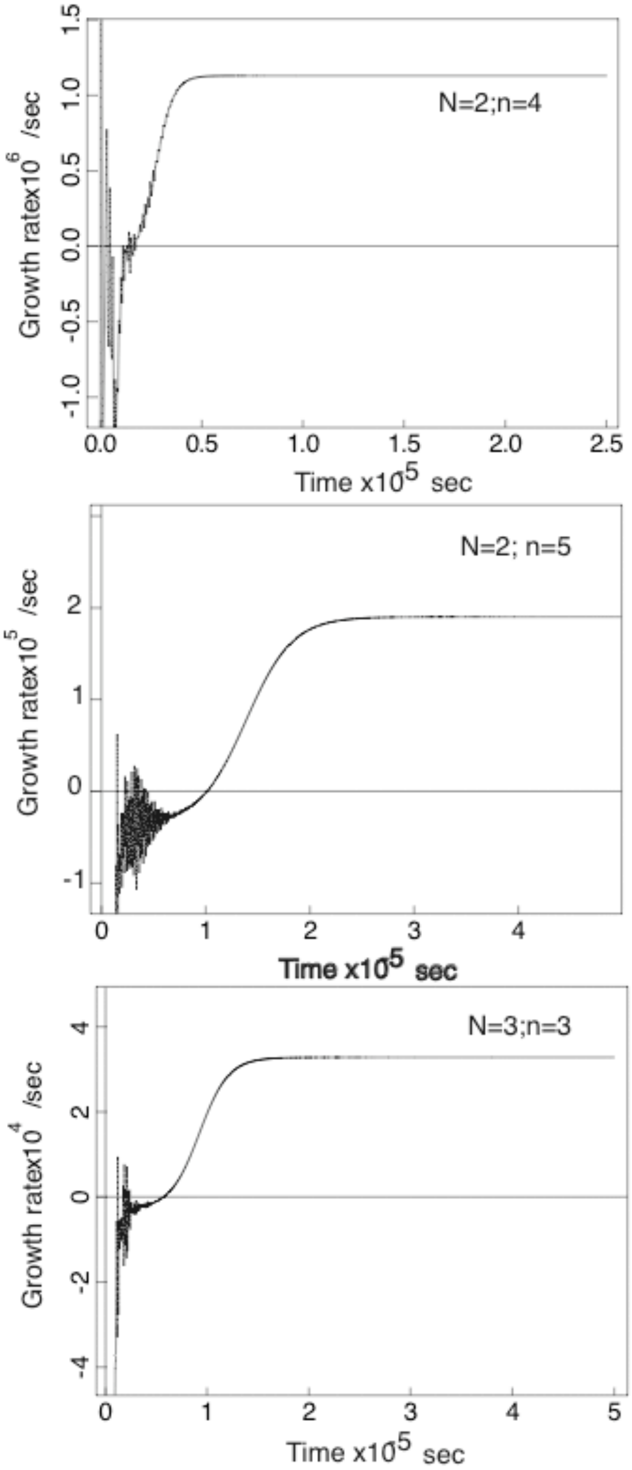

Fig.4 The growth rate results from NIMROD for N=2 and 3 showing instabilities. Constancy of growth rate shows convergence of the time dependent calculations.

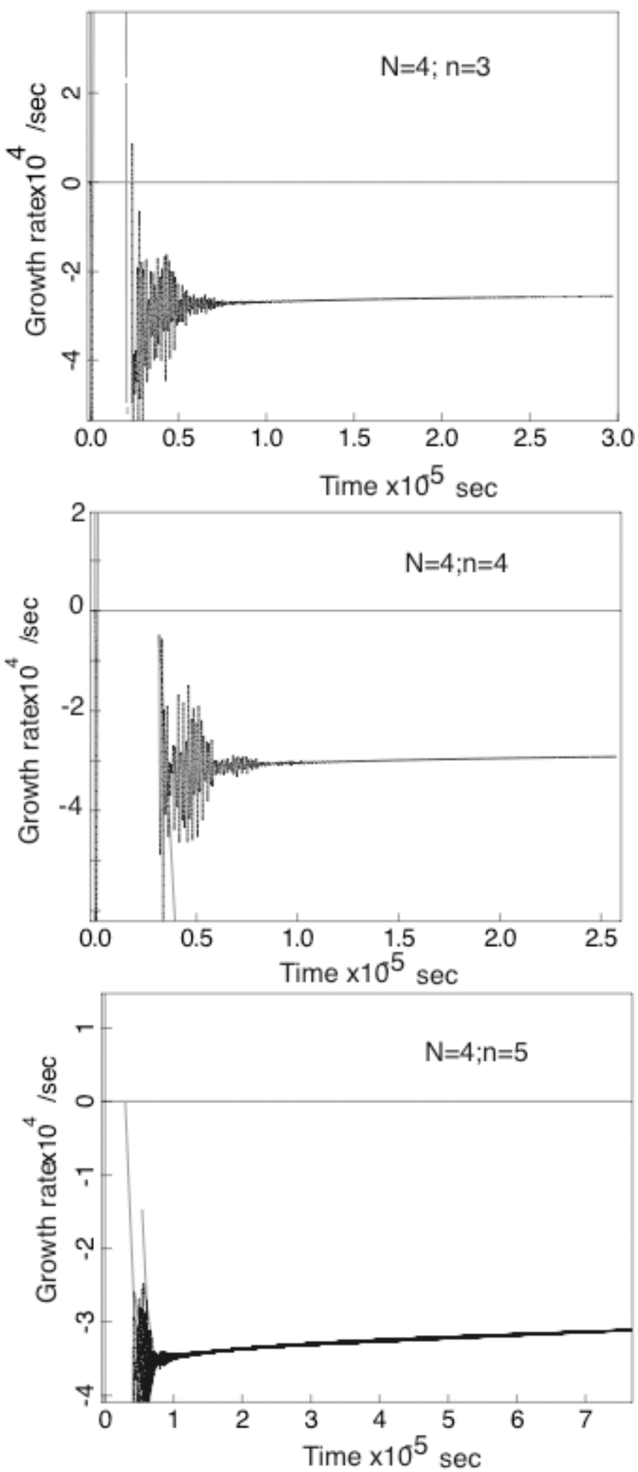

Fig.5 Growth rate (negative) for a few of the toroidal mode numbers for N=4 indicating stability.